STATISTICS AND PROBABILITY THEORY

A MATHEMATICAL APPROACH TO NATURAL FENOMENA

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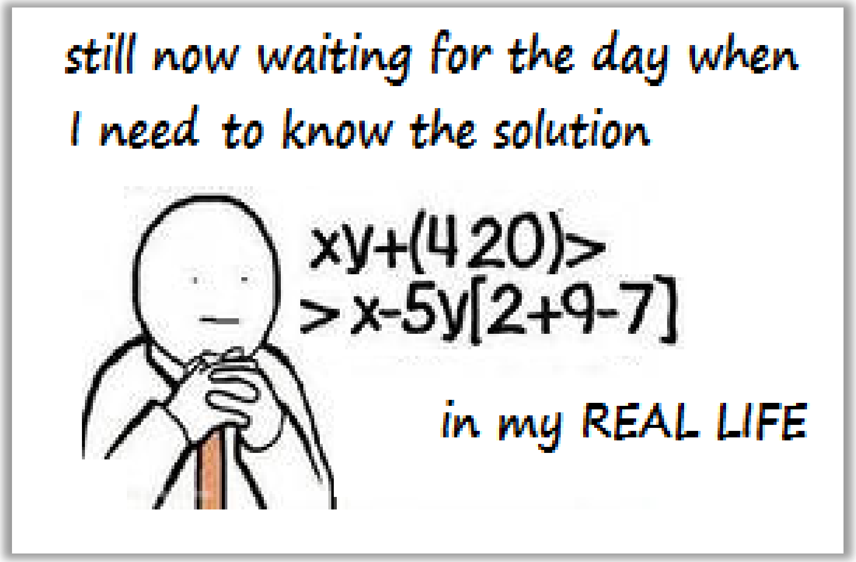
"WHAT DO WE WANT FROM THEM?"

- School education is traditionally aimed to develop ideas about hard links between phenomena.
- School mathematics remains conservative and too formal.



The negative consequences of it are felt stronger every year. Many see no use in mathematics.

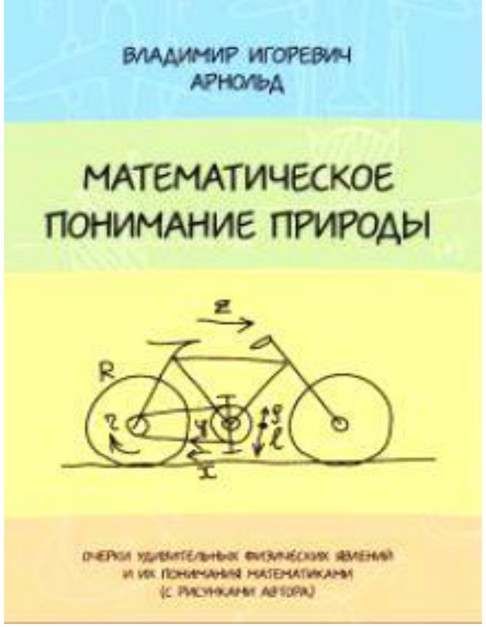
WHAT DO WE WANT FROM OURSELVES?

When we talk about the prevention and disaster preparedness in math class, we first need to clearly answer the question, what we want:

1. To study natural phenomena with mathematical methods in school;

2. To put into school math a lot of terrible and horrible examples;

3. To instill in students mathematical understanding of Nature.

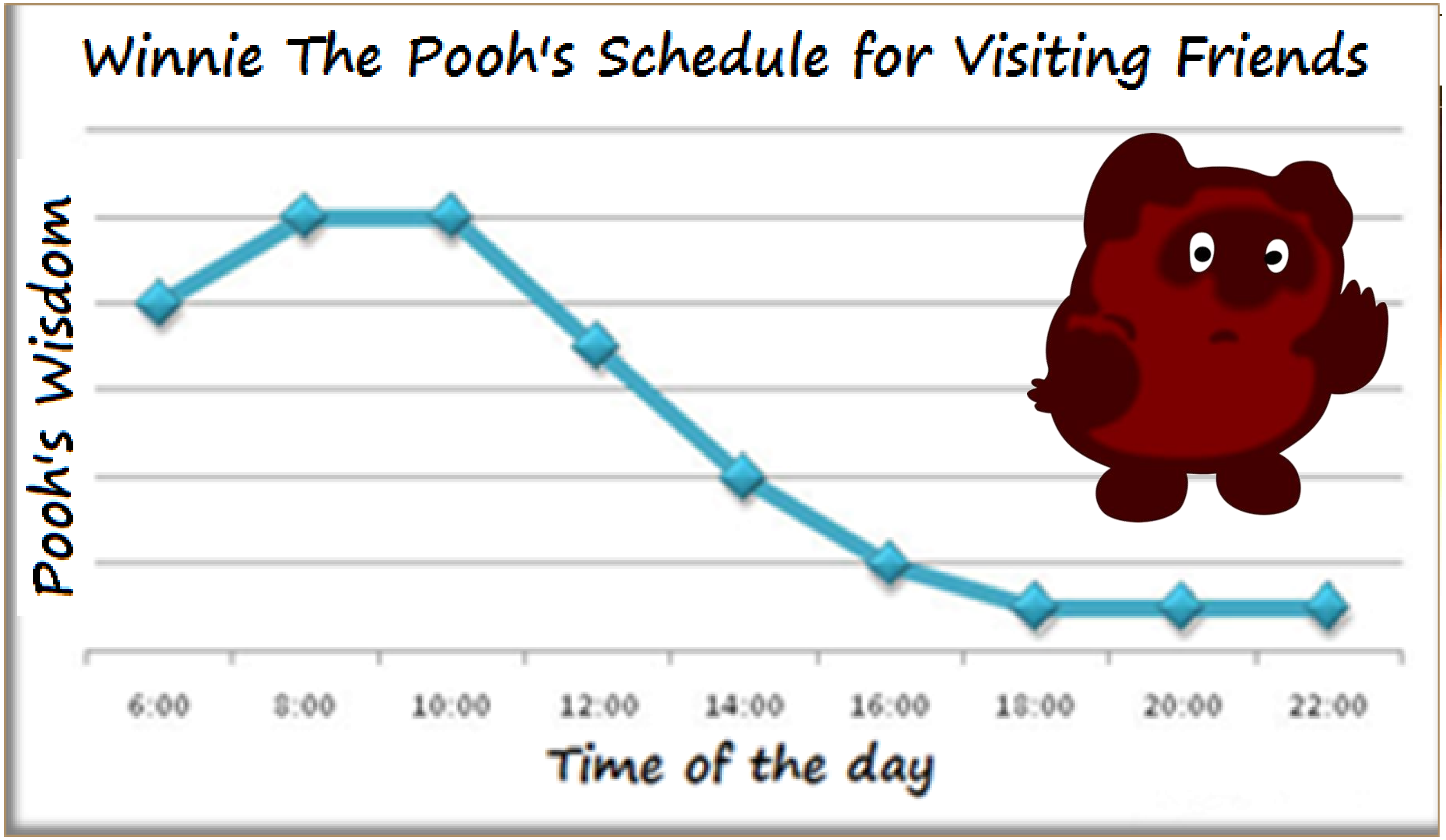


WHAT DO WE WANT FROM OURSELVES?

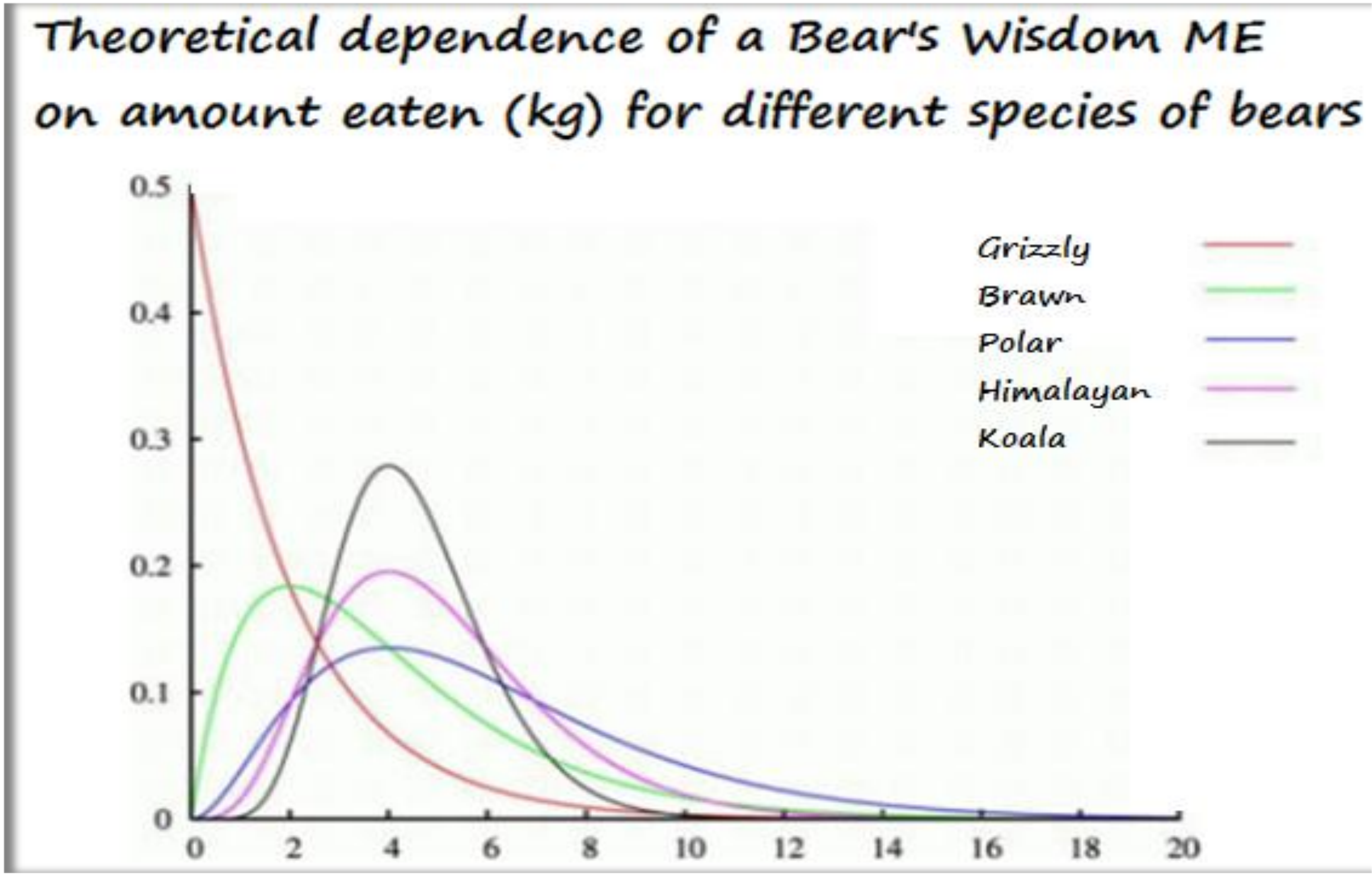
- The option 1 is not possible due to lack of knowledge and mathematical apparatus by the opinion of many sensible people
- The choice 2 makes us similar to the creators of horror movies and nothing more;
- The 3 leads us to statistics and probability, as these branches of mathematics refer directly to the nature, in particular to natural disasters.

THE PRIMACY OF STATISTICS

The statistics refers to the probability theory as well as drawing refers to the geometry.



THE PRIMACY OF STATISTICS



THE MAIN CHALLENGES

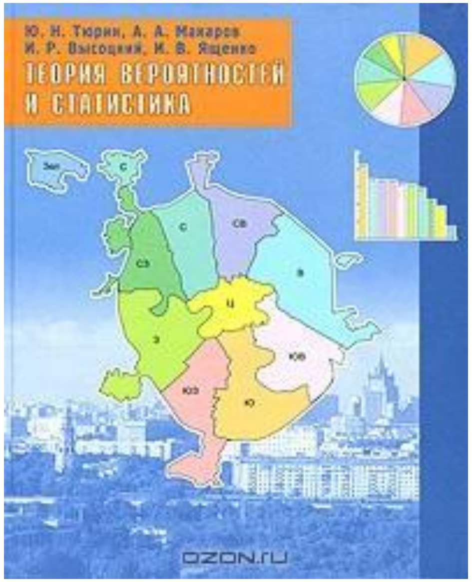
- Events are not as obvious as drawings, numbers or expressions and the chances and variability are much less intuitive than the length, area or volume.
- The second problem hides in the fact that most children under a certain age are alien to the idea of the variability and instability of phenomena.

THE MAIN AIMS

- At the minimum level we see the **need to equip students with necessary knowledge** allowing him to focus in statistical information around, to properly interpret weather forecasts, market forecast and so on.
- The second important goal is the concept of the law of large numbers and its role in a statistical sense of variable phenomena.
- At the profile level these goals are complemented by the need to acquaint the students with the basics of probabilistic modeling in simplest random experiments.

THE MAIN PRINCIPLES

- Many essential problems posed without a clear answer; situations that provoke all kinds of conjectures and imaginations.
- Providing most important and relevant topics for students



We make an emphasis on a clear understanding of variability and the law of large numbers but not on proofs and calculations. Many concepts and facts become available through the experiments, instead of using the theorems and formulas.

WHY DO WE NEED TO LEARN THE PT?

- One teacher had asked me, how to explain to a student why he or she must study this strange subject. The common answer for the question see below:

> Probabilistic and statistical methods now have deeply penetrated into math applications. They are commonly used in technics, economy, medicine and natural sciences. Especially PT becomes important relatively to development of computer technologies.

HOW TO TELL THE TRUTH IN A LUCID WAY?

There are 100 left and 100 right gloves in a box. How many gloves you must pull out by random (for you can't see which you get) to be sure to get a pair?

as the probability theory to say?

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han more than 0.9375 er four on

e ha

Not enough? Well, take…

6

$$\frac{99}{199}\cdot\frac{98}{198}\cdot\frac{97}{197}\cdot\frac{96}{196}<\left(\frac{1}{2}\right)^{4}=0{,}0625$$

HOW DOES PT WORK?

Probability theory **practically guarantees** you get the pair.

This is funny problem. See below some serious problems that also could be solved by the probability theory.

1. How many portions of chicken and fish are to be stored in the plane to satisfy all passengers almost surely?
2. What should be the minimum supply of medicines in the city in a case of a natural disaster?
3. How many emergency exits for the urgent evacuation of the shopping center needed?
4. How much should the minimum premium be in order to the insurance company will practically surely not be ruined?

HOW DOES PT WORK?

Instead of quite reliable but meaningless answers the probability theory provides reasonable and practically trustworthy answers

FORECASTS

Some family has four children, but we don't know how many boys are. We want to buy gifts for all children – car model for boys and doll houses for girls. The car and the house cost the same (1000 Baht for example). Select the best strategy – how many gifts to buy. Here we face to a simplest forecasting in an uncertain situation.

FORECAST 1

Forecast 1. Assume that they have two boys and two girls. The probability of that can be easily found with the formula or the graph. Obtain

$$P_2 = 0,375$$

If we bye two cars and two houses we pay C = 4000 but we fail at a chance of 0,625. **Our forecast is very certain but not reliable.**

FORECASTS 2 AND 3

Forecast 2. Let it be from 1 to 3 boys in the family. The probability is

$$P_{1-3} = P_1 + P_2 + P_3 = 0,25 + 0,375 + 0,25 = 0,875$$

Now we need 3 cars and 3 houses. It will cost C = 6000 but odds for the mistake turns less down to 0,125. This forecast is less certain but more reliable.

Forecast 3. We can make the least certain prognosis: from 0 to 4 boys in the family. Thus we will guess for sure: $P_{0-4} = 1$ (the probability for the mistake is 0), but it will cost us C = 8000. **Now we have absolutely reliable but most uncertain forecast we could.**

IS THE BEST SOLUTION REALLY GOOD?

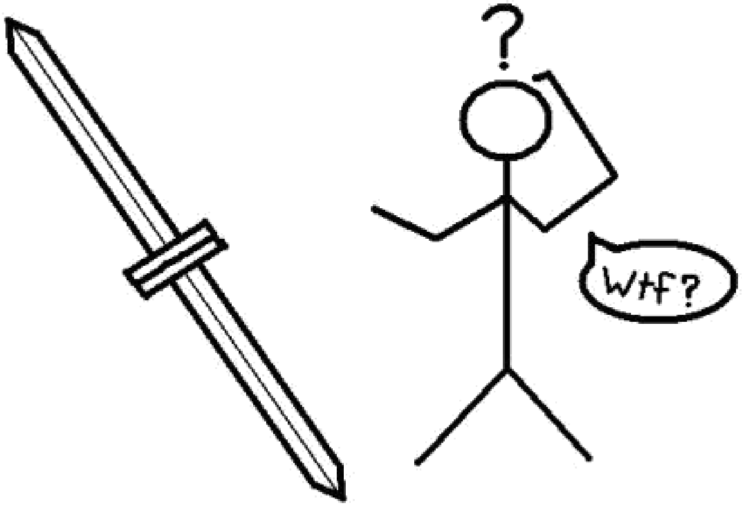
We can choose a deciding rule or a criterion. For example the criterion $Crit = \frac{P}{C} \cdot 10000$ should be maximal.

The value of *Crit* is maximal for the second forecast: from 1 to 3 boys. So we do relying on the good chances of 0,875.

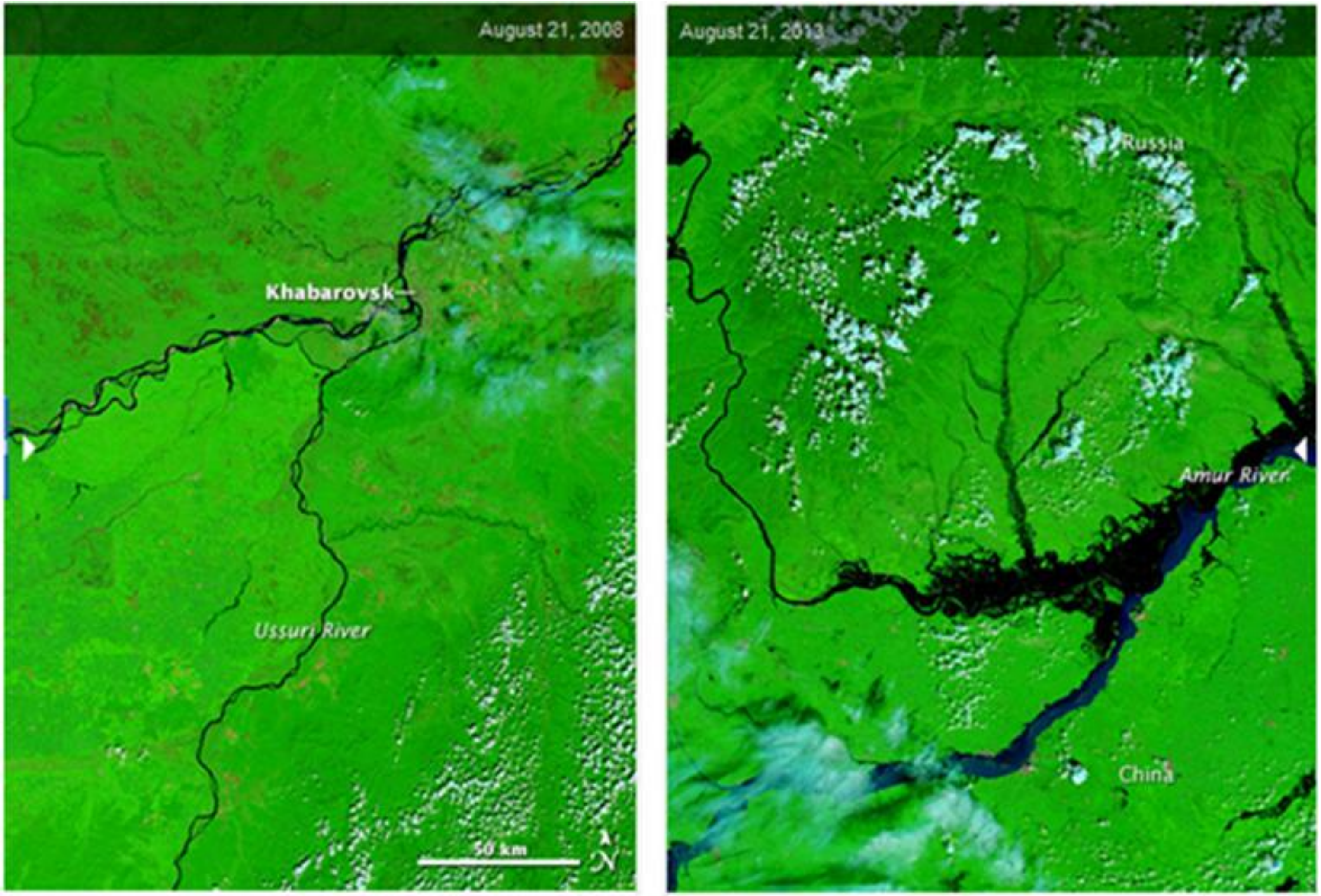
The choose of the criterion or deciding rule depends on how dangerous the mistake could be in the situation.

THE DOUBLE-EDGED SWORD

Any forecast is a "double-edged sword". The more certain the less reliable it is. In every experiment one might look for a compromise (an suitable intermediate solution) between certainty and reliability



THE FLOOD ON AMUR



PEAK WATER LEVELS AT KHABAROVSK

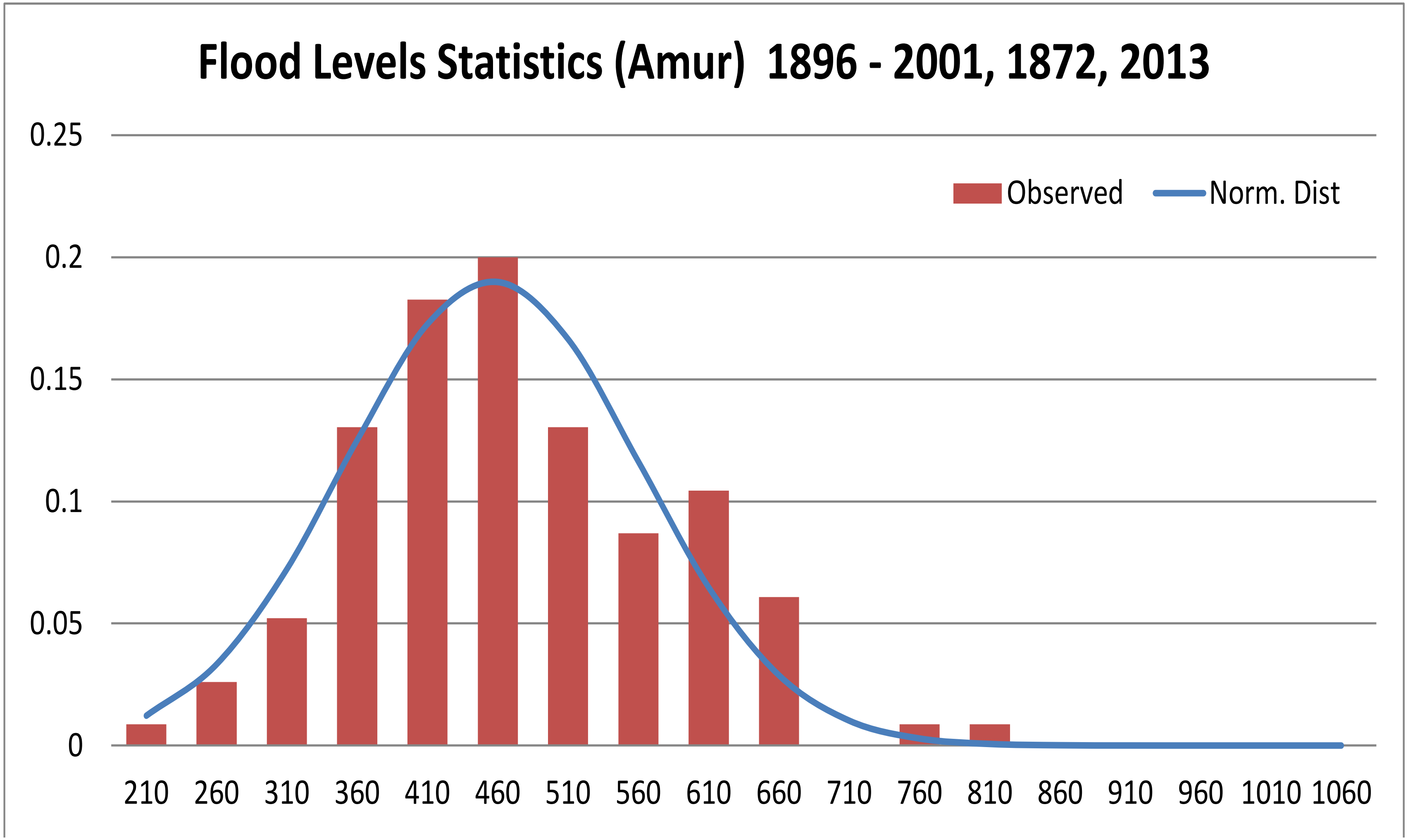
The floods at the late summer and early autumn on the Amur-river (North-East provinces of China and Russian Far East including Khabarovskiy Kray) are the common thing.

Level (cm)	210	260	310	360	410	460
Freq	0.009	0.026	0.052	0.130	0.183	0.200

Level (cm)	510	560	610	660	710	760
Freq	0.130	0.087	0.104	0.061	0.000	0.009

The data are grouped by 50 cm.

DISTRIBUTION DIAGRAM



THE FORECASTS

Forecast 1. The water will rise no higher than 460 cm. To estimate the probability of it let us sum corresponding frequencies in the lower row of the table: $\mathrm{P}(X \leq 460) \approx 0{,}6$. If the dams have height of 4.6 they will be overflowed at a chance of 0.4. This is a huge chance. Our forecast is no reliable to defend us against a flood next year.

Forecast 2:. The water will rise no higher than 660 cm. Put the frequencies together: $\mathrm{P}(X \leq 660) \approx 0{,}983$. If we built the dam of 6.6 m it will be overflowed at a probability of 0.017. This is low probability; such a flood is expected to occur no more frequently than once in 50 years. Our forecast got more reliable but less certain because the range of water level has grown.

THE FORECASTS

Forecast 3. The water might rise no more than 810 cm. The probability of that: $P(X \le 810) \approx 0{,}991$. Perhaps we can build such huge lams. They will be in use 9 times in 1000 years… Moreover, nobody can give a guarantee that water couldn't climb over some wonderful day.

THE QUESTION IS

Question: which factors we did take into account and which factors we have omitted?

The desirable answers:

- We took the statistics information for 115 years only.
- We don't know what could be before. We have a lack of data (the diagram has not regular shape that may be caused with a low number of observations).
- We don't know whether the natural conditions are changing. Maybe the global climate changing will lead to more floods in the future.

MATHEMATICS FORMALIZES IMAGES AND CONCEPTS INT… JUST IMAGES

Ma… …cription of the Nature but a
d… …n Mind.
b…
M… …’t answer for a question
ca… …t merely helps us to get to it
“W…
helps u… g…

THANK YOU MUCH
FOR YOUR KIND PATIENCE

The puppy does math.
How many images of the puppy are arisen when you look at the picture?